RISE-based trajectory tracking control of an aerial manipulator under uncertainty

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***Abstract*— This study presents a robust integral of the sign of the error (RISE)-based controller for an aerial manipulator consisting of a multi-rotor and a robotic arm which guarantees tracking error convergence to zero in the presence of uncertainties. To rigorously address underactuatedness issue, the system dynamics is decomposed into the two subsystems for which a robust controller is derived. As an intermediate result, if there exists no uncertainty, we show that the nominal closed-loop system with the proposed nominal controller is asymptotically stable without assuming that the attitude error term in the underactuated part is zero by cascaded system analysis tool. Then, a robust controller combining a nominal controller and a RISE controller is proposed and applied to both subsystems. Tracking error convergence is strictly proved through Lyapunov-based stability analysis. The performance of the controller is demonstrated in simulation with comparative studies where the proposed controller outperforms the other compared controllers in error convergence.**

***Index Terms*— Robotics, Robust control, Control applications, Aerial manipulator**

I. Introduction

An aerial manipulator, which is a robotic platform combining a multi-rotor and a robotic arm, has been a consistently studied research topic in the recent decade [1]. The combination of a multi-rotor and a robotic arm innately allows the aerial manipulator to perform various interactive tasks, by exploiting hovering ability and high mobility of a multi-rotor and dexterity of a robotic arm. Various demonstrations have been conducted including door opening [2], plug pulling [3], pick-and-place [4], and contact-based inspection [5].

Despite the aerial manipulator's vast potential, controller design has been remaining a challenging problem due to its highly complex dynamics. In contrast to a conventional multi-rotor, which is in general modeled as a single rigid body, a coupling effect from the robotic arm's motion should be suitably addressed which would otherwise incur instability [6]. To handle such issue, [7] proposes a stabilizing controller based on the dynamics described in the body coordinate, and asymptotic stability is proved using the singular perturbation theory. To handle the complexity of the aerial manipulator's dynamics, [8–10] introduce coordinate transformations through which dynamics is decoupled, and motion controllers based on such decoupled dynamics are derived. These controllers could guarantee asymptotic stability (AS), but they all assume no uncertainty nor disturbance in the model.

To consider uncertainty which prevails in actual experiments especially when interacting with an environment, various robust/adaptive controllers have been proposed. Existing robust/adaptive controllers can be classified into two categories by whether the motion of the robotic arm is regarded as disturbance or not. The first approach designs a robust/adaptive controller for a multi-rotor only, and it is more convenient than the other approach to design a controller thanks to the reduced complexity of the system model. [11, 12] utilize the multi-rotor dynamics but with additional modeling of the robotic arm's effect as a CoM shift. To guarantee boundedness of the tracking error, sliding mode control and disturbance observer are applied in [11] while [12] proposes an H_∞ controller. [13, 14] use the CoM position of the aerial manipulator instead of the multi-rotor and apply a nonlinear disturbance observer by which transient performance recovery against various uncertainties is verified using a coordinate transformation and the singular perturbation theory. Assuming that the inertia of the robotic arm is negligible, [15, 16] consider only the dynamics of the multi-rotor and the end-effector and provide a nonlinear controller which guarantees boundedness of the tracking error.

On the other hand, the second approach exploits the full knowledge of the coupled dynamics between the multi-rotor and the robotic arm and therefore, the coupling term between the two needs no longer to be deemed as disturbance. Thanks to this property, it is a merit of the second approach that better performance can be expected by virtue of the reduced source of disturbance. [17] designs a robust controller based on IDA-PBC method, but with a restriction that only a 2-dimensional quadrotor model with an n-link robotic arm is considered. [18] models the aerial manipulator using recursive Newton-Euler formulation, and designs an adaptive controller for attitude, altitude, and joint angle tracking, but without position tracking controller. [19, 20] adopt adaptive sliding mode controller and show that the system is asymptotically stable under the assumption that external disturbance is sufficiently slowly varying. [21] designs a cascade controller

Manuscript received March 21, 2022; revised May 25, 2022; accepted June 13, 2022. Date of publication XXXX XX, XXXX; date of current version XXXX XX, XXXX. This work was supported by Unmanned Vehicles Core Technology Research and Development Program through the National Research Foundation of Korea (NRF) and Unmanned Vehicle Advanced Research Center (UVARC) funded by the Ministry of Science and ICT, the Republic of Korea (2020M3C1C1A01086411). Recommended by XXXX. (Corresponding author: H. Jin Kim.)

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which consists of feedback linearization and H_∞ control and shows robustness against various types of uncertainties, but without rigorous stability analysis. [22] applies a finite-time geometric control with adaptive law which shows finite-time convergence to a small region around the origin and robustness against external disturbance. A combination of prescribed performance control and disturbance observer is presented in [23] where model/parametric uncertainties are considered as a lumped disturbance term in the model.

As discussed above, robust/adaptive control for an aerial manipulator has been an active research topic in both robotics and control communities. Among various robust/adaptive control methods, this paper aims to develop a robust integral of the sign of the error (RISE) feedback controller for the full aerial manipulator system. The RISE control, which was first proposed in [24], is a robust control technique which can guarantee asymptotic stability or exponential stability even if there exist model uncertainties and disturbance [25]. Since the controller could guarantee convergence of the tracking error to zero which was not possible in other previously mentioned controllers applied to the aerial manipulator, the RISE controller is expected to have strength in accurate manipulation. Despite such strength, to the best of the authors' knowledge, the RISE control has not been applied to an aerial manipulator as opposed to its employment in various other aerial robots [26–29].

To address underactuatedness[1] of the aerial manipulator, we first decompose the aerial manipulator dynamics into the underactuated and fully actuated subsystems with coordinate transformation. Then, a nominal controller guaranteeing asymptotic stability of the nominal system is constructed and augmented to a robust controller based on the RISE method. Tracking error convergence to zero is verified with strict Lyapunov-based stability analysis where exponential stability is proved. Comparisons with other robust controllers, adaptive sliding mode control [20] and disturbance observer-based control [14], are also conducted in simulation to validate performance of the proposed controller.

The contribution of this research can be summarized as follows:

- reformulation of the full dynamics of the aerial manipulator for the ease of controller design and analysis
- asymptotic stability analysis with thorough consideration of the underactuatedness of the nominal closed-loop aerial manipulator system
- first approach to apply RISE control to the aerial manipulator guaranteeing tracking error convergence to zero in the presence of uncertainties

A. Notations

In this work, we use I_n, $\mathtt{diag}\{a,b\}$, $\mathtt{blkdiag}\{A,B\}$ to denote an identity matrix in $\mathbb{R}^{n\times n}$, a diagonal matrix composed of scalars a,b, and a block diagonal matrix composed of matrices A and B. For a vector v and a diagonal matrix D, v_i denotes the i^{th} element of v, and D_i is the i^{th} diagonal element of D. Also, for a column vector a and b, $[a;b] := [a^\top\ b^\top]^\top$. Lastly, $c(\cdot)$, $s(\cdot)$, and $t(\cdot)$ denote shorthands for $\cos(\cdot)$, $\sin(\cdot)$, and $\tan(\cdot)$, respectively.

[1] This is a mechanical property of a multi-rotor where all 6 degrees of freedom cannot be independently controlled. This property can be observed from the fact that a multi-rotor cannot translate in x,y directions without rolling or pitching.

Fig. 1. Aerial manipulator prototype. Note that we do not assume a particular robotic arm configuration in dynamic modeling and control design.

II. Dynamic Model

Assuming a multirotor-based aerial manipulator as in Fig. 1, a control input is defined as $u = [T;\tau_\phi;\tau_\theta] \in \mathbb{R}^{4+n_\theta}$ where $T \in \mathbb{R}$, $\tau_\phi \in \mathbb{R}^3$, and $\tau_\theta \in \mathbb{R}^{n_\theta}$ are multirotor's total thrust in body z axis, multirotor's torque in body coordinate, and joint torques of the robotic manipulator. We take a generalized coordinate as $q = [p;\phi;\theta] \in \mathbb{R}^{6+n_\theta}$, where $p \in \mathbb{R}^3$, $\phi \in \mathbb{R}^3$, and $\theta \in \mathbb{R}^{n_\theta}$ are CoM position, ZYX Euler angles of a multirotor base, and joint angles of a robotic arm, respectively. n_θ denotes the number of revolute joints in the robotic arm. Regarding the aerial manipulator as a serially connected linkage where the multirotor base is considered as the 0-th link, mass and moment of inertia of the j-th link about their CoM expressed in their body frame are denoted as m_j and $\mathcal{I}_j$. For notational simplicity, we separately define configuration related to rotational motions as $r = [\phi;\theta]$, and CoM position of the aerial manipulator as $p_c \in \mathbb{R}^3$.

By solving Euler-Lagrange equation, equations of motion (EoM) can be written as

$$\begin{gathered} M\ddot{q} + C\dot{q} + G = Bu + \Delta \\ M = \begin{bmatrix} M_t & M_{tr} \\ M_{tr}^\top & M_r \end{bmatrix} \end{gathered} \tag{1}$$

where $M, C \in \mathbb{R}^{(6+n_\theta)\times(6+n_\theta)}$, $G \in \mathbb{R}^{6+n_\theta}$, $B \in \mathbb{R}^{(6+n_\theta)\times(4+n_\theta)}$ are mass matrix, Coriolis-centrifugal matrix, gravity vector, and input matrix, respectively. Particularly, input matrix can be derived as $B = \mathtt{blkdiag}\{Rb_3, Q^\top, I_{n_\theta}\}$ where $R \in \mathrm{SO}(3)$ is a rotation matrix describing orientation of the multirotor base, $b_3 = [0;0;1]$, $Q \in \mathbb{R}^{3\times3}$ is a Jacobian matrix mapping Euler angle rate $\dot{\phi}$ to body angular velocity [20]. Lastly, M_t, M_{tr}, and M_r respectively have the dimension of $M_t \in \mathbb{R}^{3\times3}, M_{tr} \in \mathbb{R}^{3\times(3+n_\theta)}, M_r \in \mathbb{R}^{(3+n_\theta)\times(3+n_\theta)}$, and Δ is external disturbance. We assume that disturbance Δ is twice differentiable, and both disturbance and its time-derivative are bounded.

Using $S \triangleq -M_t^{-1}M_{tr}$, define a new coordinate $\eta \in \mathbb{R}^{6+n_\theta}$ satisfying $\dot{q} = \tilde{S}\dot{\eta}$ where

$$\tilde{S} = \begin{bmatrix} I_3 & S \\ 0 & I_{3+n_\theta} \end{bmatrix}.$$

Then, by pre-multiplying $\tilde{S}^\top$ to (1) and using $\dot{q} = \tilde{S}\dot{\eta}$,

$$\tilde{M}\ddot{\eta} + \tilde{C}\dot{\eta} + \tilde{G} = \tilde{S}^\top Bu + \tilde{S}^\top\Delta \tag{2}$$

where $\tilde{M} = \tilde{S}^\top M\tilde{S}$, $\tilde{C} = \tilde{S}^\top(M\dot{\tilde{S}} + C\tilde{S})$, and $\tilde{G} = \tilde{S}^\top G$. Now, we state the following:

Proposition 1 ([9]): A new coordinate η satisfying $\dot{q} = \tilde{S}\dot{\eta}$ can be taken as $\eta = [p_c; r]$, and EoM is equivalent to the following decoupled dynamics

$$m_L\ddot{p}_c + m_L g b_3 = Rb_3 T + \Delta_t \tag{3a}$$
$$\tilde{M}_r(r)\ddot{r} + \tilde{C}_r(r,\dot{r})\dot{r} = \begin{bmatrix} S^\top Rb_3 & \tilde{Q}\end{bmatrix} u + \Delta_r \tag{3b}$$

where $m_L \triangleq \sum_{j=0}^{n_\theta} m_j$, $g \in \mathbb{R}$ is a gravitational acceleration constant, $\tilde{M}_r \triangleq M_r - M_{tr}^\top M_t^{-1} M_{tr}$ is symmetric, positive-definite, $\tilde{Q} \triangleq \mathtt{blkdiag}\{Q^\top, I_{n_\theta}\}$, $\Delta_t = [I_3 \quad S]\Delta \in \mathbb{R}^3, \Delta_r = [0 \quad I_{3+n_\theta}]\Delta \in \mathbb{R}^{3+n_\theta}$, and $\tilde{C}_r \in \mathbb{R}^{(3+n_\theta)\times(3+n_\theta)}$ is defined from

$$\tilde{C} = \begin{bmatrix} * & * \\ * & \tilde{C}_r \end{bmatrix}.$$

Remark 1: By *Proposition 1*, the equations of motion (1) is decomposed into translational dynamics of CoM of the aerial manipulator (3a) and the remaining rotational dynamics (3b).

III. Controller Design

A. Dynamics reformulation

In (3), compared to conventional multi-rotor dynamics [30], the body thrust T affects the rotational dynamics (3b) due to non-zero terms of $S^\top Rb_3$ in the input matrix. Therefore, if we apply an inner-loop orientation control & outer-loop translation control strategy which is widely applied for controlling multi-rotors [29, 30], the thrust T should act as disturbance to the rotational dynamics. To overcome this issue, we rearrange the dynamics (3) such that the thrust T needs not to be considered as disturbance.

Let us define $q_f = [p_{c,3}; r] \in \mathbb{R}^{4+n_\theta}$ and $q_u = [p_{c,1}; p_{c,2}] \in \mathbb{R}^2$ which represent the configuration of fully actuated, and underactuated subsystems, respectively. Then, by rearranging (3), EoM of each subsystem can be obtained as

$$m_L\ddot{q}_u = \begin{bmatrix} b_1^\top Rb_3 \\ b_2^\top Rb_3 \end{bmatrix} T + \Delta_u \tag{4a}$$
$$\begin{bmatrix} m_L & 0 \\ 0 & \tilde{M}_r \end{bmatrix}\ddot{q}_f + \begin{bmatrix} m_L g \\ \tilde{C}_r\dot{r} \end{bmatrix} = \begin{bmatrix} b_3^\top Rb_3 & 0 \\ S^\top Rb_3 & \tilde{Q} \end{bmatrix} u + \Delta_f \tag{4b}$$

where $b_1 = [1; 0; 0]$, $b_2 = [0; 1; 0]$, $\Delta_u = [\Delta_{t,1}; \Delta_{t,2}]$, and $\Delta_f = [\Delta_{t,3}; \Delta_r]$. Δ_u, Δ_f could include an error term arose by uncertainty in the CoM position. For brevity, we define $\bar{T} \triangleq b_3^\top Rb_3 T$,

$$M_f \triangleq \begin{bmatrix} m_L & 0 \\ 0 & \tilde{M}_r \end{bmatrix},\ C_f \triangleq \begin{bmatrix} m_L g \\ \tilde{C}_r\dot{r} \end{bmatrix},\ B_f \triangleq \begin{bmatrix} 1 & 0 \\ S^\top Rb_3 & \tilde{Q} \end{bmatrix},$$
$$B_u \triangleq \begin{bmatrix} b_1^\top Rb_3 \\ b_2^\top Rb_3 \end{bmatrix},\ \Psi \triangleq \begin{bmatrix} c\phi_3 & s\phi_3 \\ s\phi_3 & -c\phi_3 \end{bmatrix},\ \Phi \triangleq \begin{bmatrix} t\phi_2 \\ t\phi_1/c\phi_2 \end{bmatrix}.$$

Then, by definition, $B_u T = \Psi\Phi\bar{T}$, and (4) can be rewritten as

$$m_L\ddot{q}_u = \bar{T}\Psi\Phi_d + \bar{T}\Psi(\Phi - \Phi_d) + \Delta_u \tag{5a}$$
$$M_f\ddot{q}_f + C_f = B_f\bar{u} + \Delta_f \tag{5b}$$

where Φ_d is Φ with $\phi_{1,d}, \phi_{2,d}$ instead of ϕ_1, ϕ_2, and $\bar{u} = [\bar{T}; \tau_\phi; \tau_\theta]$. $\Phi_d \in \mathbb{R}^2$ will be used as a virtual control input to control the underactuated subsystem (5a). This cascade control approach is similar to existing works [19, 23] but differs in that the attitude error term $\bar{T}\Psi(\Phi - \Phi_d)$ is strictly considered in the controller design and analysis. To denote nominal dynamics, let $\hat{*}$ indicate $*$ with nominal physical parameters. Then, the nominal counterpart of (5) can be written as

$$\hat{m}_L\ddot{q}_u = \bar{T}\Psi\Phi_d + \bar{T}\Psi(\Phi - \Phi_d) \tag{6a}$$
$$\hat{M}_f\ddot{q}_f + \hat{C}_f = \hat{B}_f\bar{u}. \tag{6b}$$

(6) is obtained from (5) by inserting nominal parameters and canceling external disturbance. Note that the orientation error term in the underacuated dynamics $\bar{T}\Psi(\Phi - \Phi_d)$ is not treated as disturbance and is considered in the nominal dynamics.

Assumption 1: Roll and pitch angles of the multirotor base are bounded by $|\phi_1|, |\phi_2| < \pi/2$.

Assumption 2: The total thrust T is not zero.

For notational consistency and the ease of controller design, we reformulate (5) as

$$m_u\ddot{q}_u + f_u = u_u \tag{7a}$$
$$m_f\ddot{q}_f + f_f = u_f \tag{7b}$$

where

$$\begin{aligned} m_u \triangleq m_L,\ f_u \triangleq \bar{T}\Psi(\Phi_d - \Phi) - \Delta_u,\ u_u = \bar{T}\Psi\Phi_d \\ m_f \triangleq B_f^{-1}M_f,\ f_f \triangleq B_f^{-1}(C_f - \Delta_f),\ u_f = \bar{u}. \end{aligned} \tag{8}$$

Thanks to *Assumption 1*, B_f^{-1} always exists and therefore, all variables in (8) are well-defined. Note also that with *Assumption 1, 2*, a bijective map between $u_u \in \mathbb{R}^2$ and $\phi_{1,d}, \phi_{2,d} \in (-\pi/2, \pi/2)$ can be obtained as

$$\begin{aligned} \phi_{2,d} &= \tan^{-1}\left(\tfrac{1}{T}\left(c\phi_3 u_{u,1} + s\phi_3 u_{u,2}\right)\right), \\ \phi_{1,d} &= \tan^{-1}\left(\tfrac{c\phi_{2,d}}{T}\left(s\phi_3 u_{u,1} - c\phi_3 u_{u,2}\right)\right). \end{aligned} \tag{9}$$

For controller design, we combine control inputs of the nominal controller and robust controller as $u_u = u_{u,n} + u_{u,r}$, $u_f = u_{f,n} + u_{f,r}$ where $u_{u,n}, u_{f,n}$ are nominal inputs which stabilize the nominal system, and $u_{u,r}, u_{f,r}$ are robust control inputs compensating model uncertainties and disturbance.

B. Nominal control design

In theory, RISE control [24] could guarantee AS even without a nominal controller. However, since RISE control relies solely on integrals of error signals, transient performance could be degraded particularly for hovering control where unceasing non-zero thrust is required. Hence, for better transient performance, we design a nominal controller as in [26, 28] for both underactuated and fully actuated subsystems, which could guarantee AS of the nominal system.

The nominal controller is designed with $\hat{m}_u \triangleq \hat{m}_L$, $\hat{m}_f \triangleq \hat{B}_f^{-1}\hat{M}_f$, and $\hat{f}_f \triangleq \hat{B}_f^{-1}\hat{C}_f$ as follows:

$$u_{u,n} = \hat{f}_u + \hat{m}_u(\ddot{q}_{u,d} + K_{ud}\dot{e}_{u1} + K_{up}e_{u1}) \tag{10a}$$
$$u_{f,n} = \hat{f}_f + \hat{m}_f(\ddot{q}_{f,d} + K_{fd}\dot{e}_{f1} + K_{fp}e_{f1}) \tag{10b}$$

where $e_{u1} = q_{u,d} - q_u$, $e_{f1} = q_{f,d} - q_f$. $K_{up}, K_{ud} \in \mathbb{R}^{2\times 2}$ and $K_{fp}, K_{fd} \in \mathbb{R}^{(4+n_\theta)\times(4+n_\theta)}$ are control gain matrices. For control structure uniformity, $\hat{f}_u = 0$ is added in (10a).

C. RISE control design

Similar to [28, 29], we design a RISE controller for both the underactuated and fully actuated parts. With control gains $\rho_u \in \mathbb{R}, K_u, \Lambda_{u1}, \Lambda_{u2}, \Lambda_{u3} \in \mathbb{R}^{2\times 2}$ and $\rho_f \in \mathbb{R}, K_u, \Lambda_{f1}, \Lambda_{f2}, \Lambda_{f3} \in \mathbb{R}^{(4+n_\theta)\times(4+n_\theta)}$, robust feedback control inputs based on RISE are defined as

$$u_{u,r} = (K_u + \rho_u I_2)(e_{u2}(t) - e_{u2}(0)) + \int_0^t (K_u + \rho_u I_2)\Lambda_{u2} e_{u2}(\tau) + \Lambda_{u3}\mathrm{sgn}(e_{u2}(\tau))d\tau \tag{11}$$

$$u_{f,r} = (K_f + \rho_f I_{4+n_\theta})(e_{f2}(t) - e_{f2}(0)) + \int_0^t (K_f + \rho_f I_{4+n_\theta})\Lambda_{f2} e_{f2}(\tau) + \Lambda_{f3}\mathrm{sgn}(e_{f2}(\tau))d\tau \tag{12}$$

where $e_{u2} = \dot{e}_{u1} + \Lambda_{u1}e_{u1}$, $e_{f2} = \dot{e}_{f1} + \Lambda_{f1}e_{f1}$, and $\mathrm{sgn}(\cdot)$ is a sign function.

IV. Stability Analysis

A. Nominal closed-loop system analysis

We first show that the nominal dynamics with nominal controller satisfies AS. Analysis of the nominal closed-loop system is presented after the following lemma.

Lemma 1 ([7, 31]): Consider a system

$$\dot{x}_1 = f_1(x_1) \tag{13a}$$

$$\dot{x}_2 = f_2(x_1, x_2) \tag{13b}$$

where $x_1, f_1 \in \mathbb{R}^{n_1}$ and $x_2, f_2 \in \mathbb{R}^{n_2}$. If $x_1 = 0$ is AS for $\dot{x}_1 = f_1(x_1)$, and $x_2 = 0$ is AS for $\dot{x}_2 = f_2(0, x_2)$, then (13) is AS to $x_1 = 0$ and $x_2 = 0$.

Theorem 1: For positive-definite, diagonal matrices K_{up}, K_{ud}, K_{fp}, K_{fd}, the nominal closed-loop system consisting of EoM (6), control input (10) ($u_u = u_{u,n}$, $u_f = u_{f,n}$), and roll, pitch desired values (9) is AS.

Proof: The nominal closed-loop system can be written as

$$\ddot{e}_{u1} + K_{ud}\dot{e}_{u1} + K_{up}e_{u1} = \bar{T}\Psi(\Phi_d - \Phi)/\hat{m}_L, \tag{14a}$$

$$\ddot{e}_{f1} + K_{fd}\dot{e}_{f1} + K_{fp}e_{f1} = 0. \tag{14b}$$

Then, since control gains are positive-definite, diagonal matrices, (14b) is AS to $[e_{f1}; \dot{e}_{f1}] = 0$, and (14a) is AS to $[e_{u1}; \dot{e}_{u1}] = 0$ assuming $[e_{f1}; \dot{e}_{f1}] = 0$. Therefore, by *Lemma 1*, the closed-loop system (14) is AS. ■

Remark 2: Theorem 1 shows that even in the presence of attitude error term $\bar{T}\Psi(\Phi - \Phi_d)$ in translational dynamics which inevitably occurs due from underactuatedness, AS can still be obtained.

B. Actual closed-loop system analysis

Since the system (7) and controllers (10), (11), (12) of the underactuated and fully actuated part share the same structure, we first analyze the following virtual system and control input:

$$m\ddot{q} = u - f \tag{15a}$$

$$u_n = \hat{f} + \hat{m}(\ddot{q}_d + K_d\dot{e}_1 + K_p e_1) \tag{15b}$$

$$u_r = (K + \rho I_n)(e_2(t) - e_2(0)) + \int_0^t (K + \rho I_n)\Lambda_2 e_2(\tau) + \Lambda_3\mathrm{sgn}(e_2(\tau))d\tau \tag{15c}$$

where $[q; \dot{q}] \in \mathbb{R}^{2n}$ is a system state, $u = u_n + u_r \in \mathbb{R}^n$ is a control input, and $\rho \in \mathbb{R}, K_p, K_d, K, \Lambda_1, \Lambda_2, \Lambda_3 \in \mathbb{R}^{n\times n}$ are diagonal control gain matrices. $e_1 = q_d - q$, $e_2 = \dot{e}_1 + \Lambda_1 e_1$. Note that the subscripts u, f are omitted in the virtual system (15) to differentiate it from the actual system (7).

For the analysis, first define $e_3 = \dot{e}_2 + \Lambda_2 e_2$. Then,

$$m\dot{e}_3 = N - (K + \rho I_n)e_3 - \Lambda_3\mathrm{sgn}(e_2) - \frac{1}{2}\dot{m}e_3 - e_2 \tag{16}$$

where

$$N = (\dot{m}\ddot{q} - \dot{\hat{m}}\ddot{q}_d) - \dot{\hat{m}}(K_d\dot{e}_1 + K_p e_1) - \hat{m}(K_d\ddot{e}_1 + K_p\dot{e}_1) + (\dot{f} - \dot{\hat{f}}) + (m - \hat{m})\dddot{q}_d + m(\Lambda_1\ddot{e}_1 + \Lambda_2\dot{e}_2) + \frac{1}{2}\dot{m}e_3 + e_2.$$

Let N_d be defined as

$$N_d = (\dot{m}_d - \dot{\hat{m}}_d)\ddot{q}_d + (\dot{f}_d - \dot{\hat{f}}_d) + (m_d - \hat{m}_d)\dddot{q}_d$$

where we overload the notation $*_d$ to denote $*$ with the desired state $[q_d; \dot{q}_d]$. For f_u and f_f, $f_{u,d} = -\Delta_u$, $f_{f,d} = B_{f,d}^{-1}(C_{f,d} - \Delta_f)$. Then, as in *Remark 3* of [24], the following can be obtained for $\tilde{N} = N - N_d$ as

$$\|\tilde{N}\| \leq \mu(\|e\|)\|e\| \tag{17}$$

where $e = [e_1; e_2; e_3]$ and $\mu(\cdot)$ is a positive, globally invertible, non-decreasing function.

Lemma 2 ([27]): Take $\Lambda_{3,i} > \|N_{d,i}\|_\infty + \Lambda_{2,i}^{-1}\|\dot{N}_{d,i}\|_\infty$ $\forall i = 1, \cdots, n$. Then, the following function

$$Q(t) = \int_0^t 2\dot{e}_2^\top \Lambda_3\mathrm{sgn}(e_2)d\tau + 2\sum_{i=1}^n \Lambda_{3,i}|e_{2,i}(0)| - N_d^\top e_2$$

satisfies $0 \leq Q(t) \leq \sum_{i=1}^n (2\Lambda_{3,i} + \|N_{d,i}\|_\infty)|e_{2,i}(t)|$.

Theorem 2: For the positive-definite, diagonal control gains satisfying $2\Lambda_2 > I_n$, the closed-loop system of (15) is semi-globally exponentially stable.

Proof: Consider the following Lyapunov candidate function:

$$V = \frac{1}{2}e_1^\top e_1 + \frac{1}{2}e_2^\top e_2 + \frac{1}{2}e_3^\top m e_3 + Q. \tag{18}$$

Then, by *Lemma 2*, V satisfies the lower and upper bounds of

$$\gamma_1\|e\|^2 \leq V \leq \gamma_2\|e\|^2 + \sum_{i=1}^n (2\Lambda_{3,i} + \|N_{d,i}\|_\infty)|e_{2,i}(t)|$$

where $\gamma_1 = \frac{1}{2}\min\{1, \underline{m}\}$ and $\gamma_2 = \frac{1}{2}\max\{1, \overline{m}\}$. Positive constants $\underline{m}, \overline{m}$ are defined to satisfy $\underline{m} \leq m \leq \overline{m}$. Computing the time derivative of V,

$$\dot{V} \leq -e_1^\top\Lambda_1 e_1 - e_2^\top\Lambda_2 e_2 + e_2^\top e_1 - e_3^\top(K + \rho I_n)e_3 + e_3^\top\tilde{N} - \sum_{i=1}^n \Lambda_{2,i}\Lambda_{3,i}|e_{2,i}| + \mathcal{O}(t)$$

where $\mathcal{O}(t) = -e_2^\top(\dot{N}_d - \Lambda_2 N_d) - \sum_{i=1}^n \Lambda_{2,i}\Lambda_{3,i}|e_{2,i}|$. From the condition on Λ_3 presented in *Lemma 2*, $\mathcal{O}(t) \leq 0$, and by additionally applying (17),

$$\dot{V} \leq -(\lambda^* - \frac{\mu^2(\|e\|)}{4k^*})\|e\|^2 - \sum_{i=1}^n \Lambda_{2,i}\Lambda_{3,i}|e_{2,i}|$$

where we used $e_2^\top e_1 \leq \frac{1}{2}(\|e_2\|^2 + \|e_1\|^2)$. Therefore, there exists a positive constant c satisfying $\dot{V} \leq -cV$

TABLE I

PHYSICAL PARAMETERS AND CONTROLLER GAINS IN SIMULATION (DIAGONAL ELEMENTS FOR MATRICES)

Parameter	Value (kg)	Parameter	Value (kgm^2)
$\hat{m}_0$	2.4	$\hat{\mathcal{I}}_0$	$10^{-2}[2.5, 2.5, 5.0]$
$\hat{m}_1$	0.41	$\hat{\mathcal{I}}_1$	$10^{-2}[0.5, 1.0, 1.0]$
$\hat{m}_2$	0.16	$\hat{\mathcal{I}}_2$	$10^{-4}[0.0, 2.3, 2.3]$
Gain	Value	Gain	Value
K_{up}	$[6, 6]$	K_{fp}	$10^2[1, 4.8, 4.8, 8, 5, 5]$
K_{ud}	$[4, 4]$	K_{fd}	$10^1[5, 4, 4, 4, 10, 10]$
K_u, ρ_u	$[0.5, 0.5], 0.1$	K_f, ρ_f	$10^{-1}[1, 6, 6, 6, 1, 1], 0.05$
Λ_{u1}	$[20, 20]$	Λ_{f1}	$[2, 45, 45, 45, 1.5, 1.5]$
Λ_{u2}	$[3, 3]$	Λ_{f2}	$10^{-1}[30, 8, 8, 8, 1, 1]$
Λ_{u3}	$[0.1, 0.1]$	Λ_{f3}	$10^{-2}[100, 4, 4, 4, 1, 1]$

for $\|e\| < \mu^{-1}(2\sqrt{k^*\lambda^*})$ where $k^* = \min_i(K_i)$, $\lambda^* = \min\{\frac{1}{2}\min_i(\Lambda_{1,i}), \min_i(\Lambda_{2,i}) - \frac{1}{2}, \rho\}$. Thus, by referring to the process in [24], the region of attraction $\mathcal{S} := \{e \in \mathcal{D} | W(e) < \gamma_1(\mu^{-1}(2\sqrt{k^*\lambda^*}))^2\}$ can be obtained where $W(e) := \gamma_2\|e\|^2 + \sum_{i=1}^{n}(2\Lambda_{3,i} + \|N_{d,i}\|_\infty)|e_{2,i}(t)|$ and $\mathcal{D} := \{e \in \mathbb{R}^{2n} | \|e\| < \mu^{-1}(2\sqrt{k^*\lambda^*})\}$, and this completes the proof. ■

Remark 3: Rigorous treatment for the discontinuous right hand side of $\dot{V}$ due to $\dot{e}_3$ (16), which is based on Filippov solution, can be found in [27]. Note that similar Lyapunov-based closed-loop system anaylsis was conducted in the paper, and the analysis can also be applied to validate current development.

Remark 4: By referring to *Lemma 2* and the definition of N_d, it is noteworthy that the nominal controller contributes to reducing the required lower bound of the control gain Λ_3.

Theorem 3: Given that external disturbances Δ_u, Δ_f and their time derivatives are bounded, with proper control gains of both underactuated and fully actuated subsystems satisfying premises of *Theorem 1*, *Lemma 2*, and *Theorem 2*, the closed-loop system composed of (7), (10), (11), and (12) is exponentially stable.

Proof: Constructing a candidate Lyapunov function $V_a = V_u + V_f$ where V_u, V_f are Lyapunov functions of underactuated and fully actuated parts defined in the same way as (18), the stability result in *Theorem 3* can be obtained by the same procedure presented in the proof of *Theorem 2*. Note that although (17) for the underactuated subsystem should be modified to $\|\tilde{N}_u\| \le \mu_{u1}(\|e_u\|)\|e_u\| + \mu_{u2}(\|e_f\|)\|e_f\|$ due to the orientation error term $\bar{T}\Psi(\Phi_d - \Phi)$ in f_u, this does not limit the stability analysis since there exists a positive, globally invertible, non-decreasing function $\mu_a(\cdot)$ satisfying $\|\tilde{N}_u\| \le \mu_a(\|e_a\|)\|e_a\|$ where $e_a = [e_u; e_f]$. e_u, e_f are defined in the same way as e appeared in (17). ■

Remark 5: Due to the use of Euler angles, which suffers from singularity, only local result is acquired in *Theorem 3*. However, note that within the singularity-free region $\mathcal{F} := \{e_a \in \mathbb{R}^{12+2n_\theta} | |\phi_1|, |\phi_2| < \pi/2\}$, the region of attraction appeared in *Theorem 2* can be expanded by taking larger control gains to increase k^*, λ^*.

V. SIMULATION RESULTS

To validate the proposed controller, we perform two different simulations where the aerial manipulator is considered to have a 2 degrees of freedom robotic arm. Nominal inertial

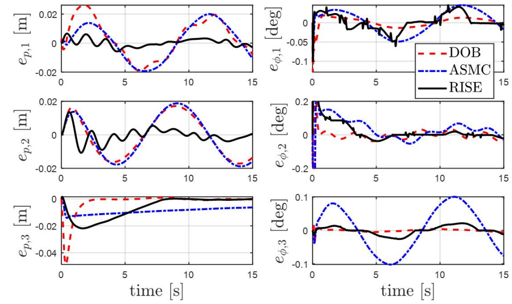

Fig. 2. Tracking error plot of scenario 1. $e_p := p - p_d$, $e_\phi := \phi - \phi_d$.

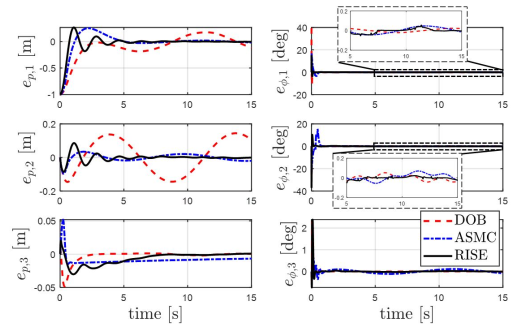

Fig. 3. Tracking error plot of scenario 2. $e_p := p - p_d$, $e_\phi := \phi - \phi_d$.

parameters and gains of the proposed controller used in simulation can be found in Tab. 1 where only the diagonal elements are listed for matrix-valued parameters/gains. Definitions of $m_j, \mathcal{I}_j$ can be found in II.

The first scenario is to regulate position deviation from zero where $p_d = [0; 0; 0]$ for all time, while the robotic arm is commanded to oscillate as $\theta_d = [\frac{\pi}{4}\cos(\frac{\pi}{5}t); \frac{\pi}{4}\sin(\frac{\pi}{5}t))]$. On the other hand, in the second scenario, the aerial manipulator is commanded to follow a circular trajectory as $p_d = [\cos(\frac{\pi}{5}t); \sin(\frac{\pi}{5}t); 0]$ in the presence of robotic arm's oscillation as $\theta_d = [\frac{\pi}{4}\cos(\frac{\pi}{5}t); \frac{\pi}{4}\sin(\frac{\pi}{5}t))]$. The desired yaw angle is defined to be 0 in both scenarios, for all time. To test robustness against model uncertainty and external disturbance, in both scenarios, 30% uncertainty is applied to all inertial parameters as $m_j = 1.3\hat{m}_j$, $\mathcal{I}_j = 1.3\hat{\mathcal{I}}_j$ $\forall j = 0, 1, 2$. Time-varying external disturbance is applied as $\Delta = [\Delta_a; \Delta_b]$ where $\Delta_a = [\sin(\frac{\pi}{5}t); \cos(\frac{\pi}{5}t); 1]$, $\Delta_b = 0.1\sin(\frac{\pi}{5}t)[1; 1; 1; 0.1; 0.1]$ in both scenarios.

For comparison, we choose two robust controllers from each category discussed in I, which are disturbance observer (DOB)-based controller [14] and adaptive sliding mode controller (ASMC) [20]. Due to the page limit, only tracking errors of position p and orientation ϕ of the multi-rotor is illustrated in Figs. 2, 3. The left columns of Figs. 2, 3 denote x,y,z directional position tracking error from the top to the bottom, and the right columns denote roll, pitch, yaw orientation tracking error, again from the top to the bottom. Traversed position trajectories during both scenarios can be found in Fig. 4. The proposed controller is plotted in black dashed lines and denoted with the legend RISE while DOB and ASMC controllers are plotted in red dotted lines and blue dash-single dotted lines, respectively. In all simulations, as can be observed in Figs. 2, 3, 4, the proposed RISE method outperforms the other comparing two in error convergence. In the error bound perspective however, DOB shows better performance in attitude tracking, and this results from the fact

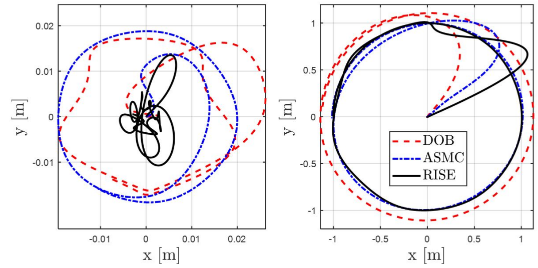

Fig. 4. Position trajectories traversed during the operation time of 15 seconds. The left figure shows the result of scenario 1 and the right figure is the one from scenario 2. The objective of scenario 1 in the left is to regulate the position at the origin while that of scenario 2 in the right is to track a circular trajectory of radius 1 centered at the origin.

that DOB [14] can guarantee the boundedness of error during the transient response, though not the asymptotic convergence. Note that each controller utilizes the same control gains in both scenarios, and sufficiently high gains are selected in all controllers to show comparable performance to the other two controllers. For each controller, a uniformly positive total thrust T is obtained in both scenarios.

VI. Conclusion

In this paper, we presented a RISE-based controller for an aerial manipulator which guarantees exponential stability in the presence of uncertainties. To formally consider the underatuatedness issue of the aerial manipulator, dynamics decomposition into the two underactuated and fully actuated subsystems was proposed on which a robust controller was designed. We first showed that the nominal feedback controller could guarantee asymptotoic tracking if there exists no uncertainty. A robust controller was then proposed by combining a nominal controller and a RISE controller, applied to both subsystems. Through Lyapunov-based stability analysis, we proved that the tracking error converges to zero exponentially, and comparative studies with existing robust controllers for an aerial manipulator demonstrated that the proposed method surpassed the others in convergence property.

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